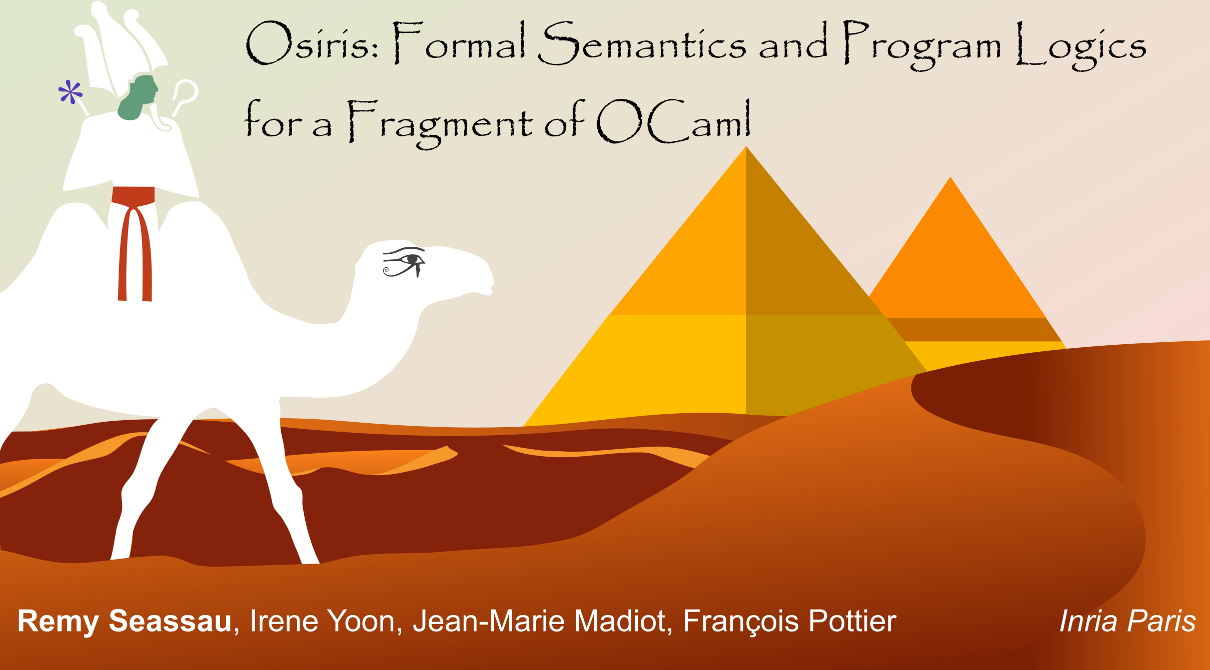
Osiris: Formal Semantics and Program Logics for a Fragment of OCaml



Remy Seassau, Irene Yoon, Jean-Marie Madiot, François Pottier

Inria Paris

The Osiris Project:

- No object system
- No formalisation of the type system

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Everything else is fair game

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This talk:

- Sequential OCaml 5
- Effect Handlers

What is Osiris?

ROCQ

- src
  - alpha_renaming.ml
  - base_tactics.ml
  - beta_red.ml
  - environment.ml
  - inductive.ml
  - instructionHandler.ml
  - substitution.ml
  - typing.ml
  - dune

Automatically generate

- code
  - og_alpha_renaming.v
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Prove properties about

Iris

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OCaml Has No Formal Semantics

"This document is intended as a reference manual for the OCaml language. [...] No attempt has been made at mathematical rigor: words are employed with their intuitive meaning, without further definition."

OCaml Has No Formal Semantics (yet!)

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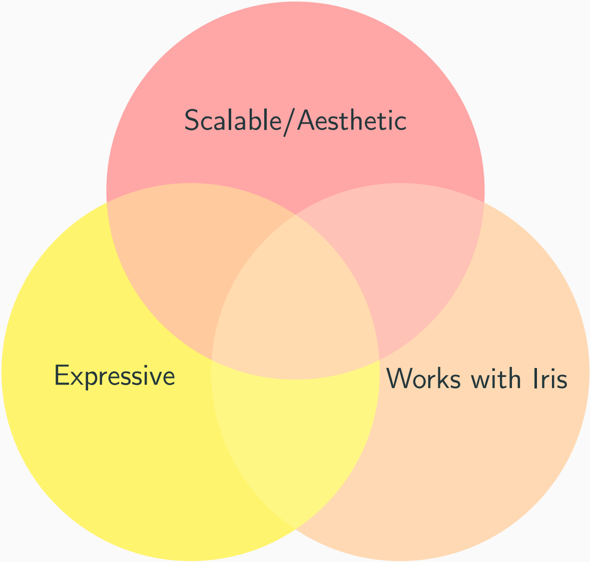
Goals:

- Design and implement formal, mechanized semantics for OCaml
- Build a framework to reason about these semantics

Design Constraints



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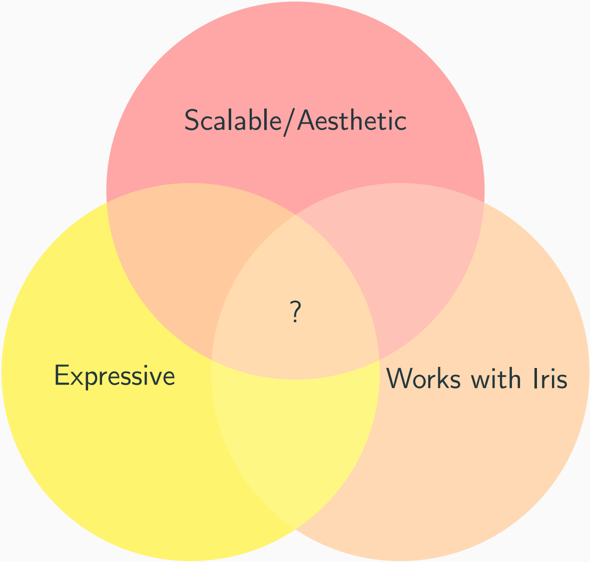

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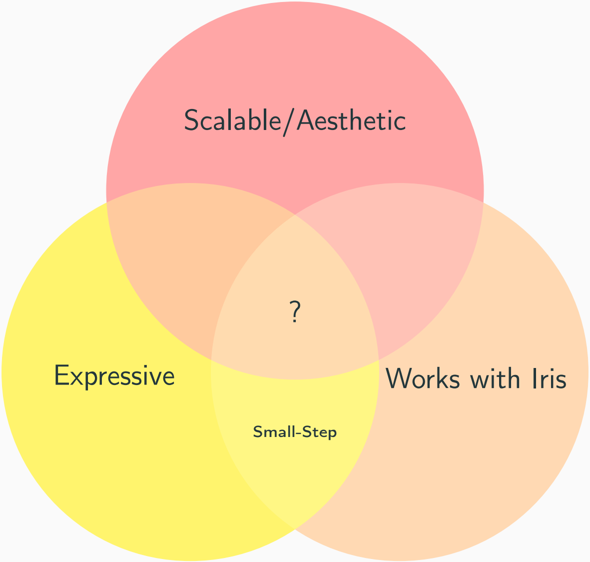

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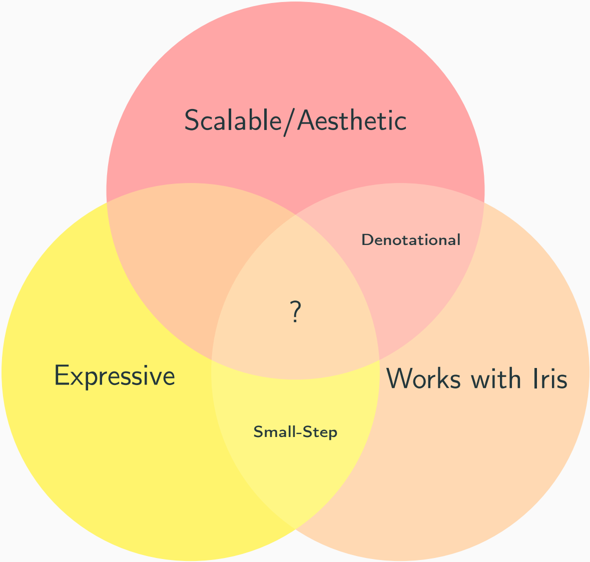

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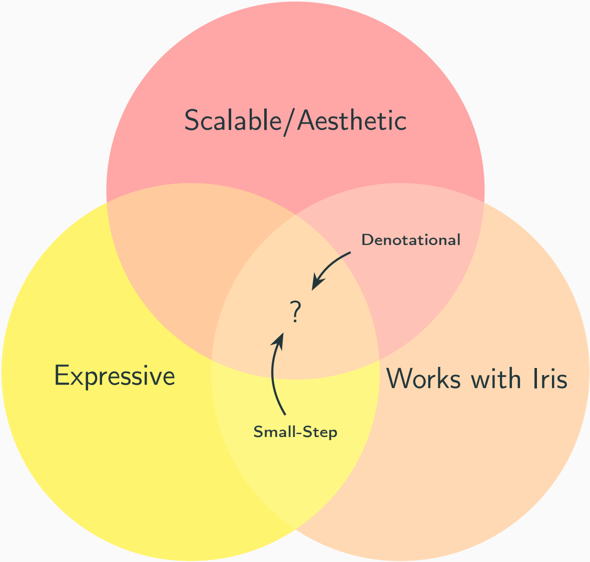

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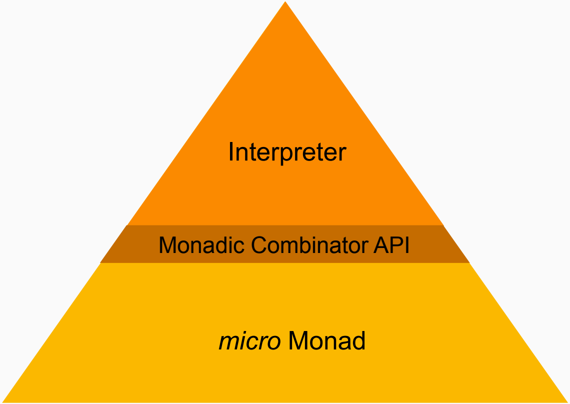
A Layered Monadic Definitional Interpreter



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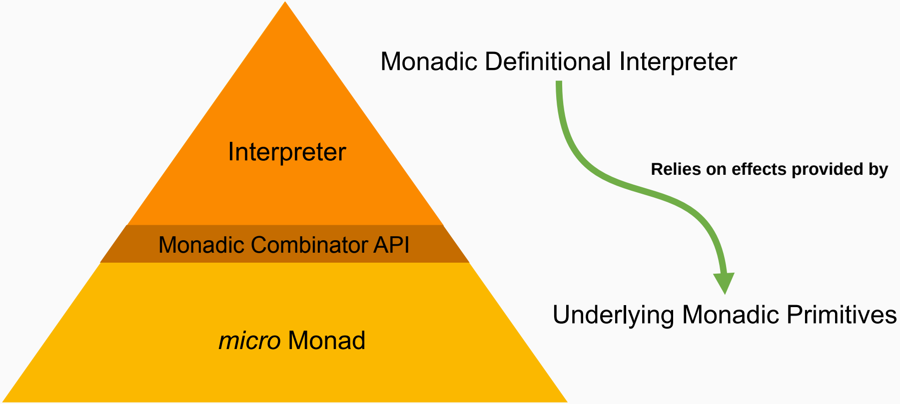

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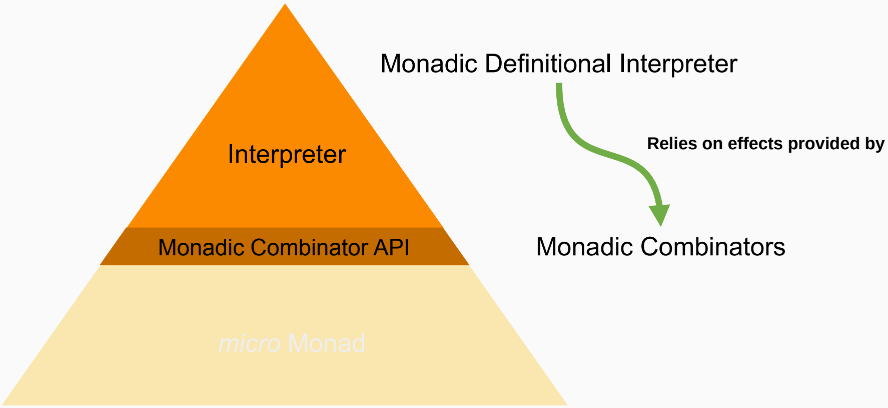

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Fixpoint eval η e : micro val exn :=
  match e with
  | ...
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The top-level interpreter has type:

$$\mathit{eval} : \mathit{env} \to \mathit{expr} \to \mathit{micro}\ \mathit{val}\ \mathit{exn}$$

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The top-level interpreter has type:

$$\textit{eval} : \textit{env} \to \textit{expr} \to \textit{micro val exn}$$

```
Inductive val :=
  | VInt (i : int) | VString (s: string)
  | VTuple (vs : list val)
  | VData (c : data) (v : list val)
  | VClo (η : env) (x : var) (e : expr)
  | VCont (l : loc)
  | ...
```

```
Fixpoint eval η e : micro val exn :=
  match e with
  | ...
  | EApp e1 e2 ⇒
      '((η, x, e), v2) ← par (as_clo (eval η e1)) (eval η e2) ;
      eval ((x, v2) :: η) e
  | ...
```

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Fixpoint eval η e : micro val exn :=
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      please_eval ((x, v2) :: η) e
  | ...
```

```
perform e

match e with
| p₁ -> e₁
| exception p₂ -> e₂
| effect p₃, k -> e₃
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Fixpoint eval η e :=
  | ...
  | EPerform e =>
      eff <- eval η e ;
      perform eff
  | EMatch e bs =>
      handle (eval η e) (fun o => eval_branches η o bs)
```

$\boxed{\text{O3Ret v} \mid \text{O3Throw exn} \mid \text{O3Perform k v}}$

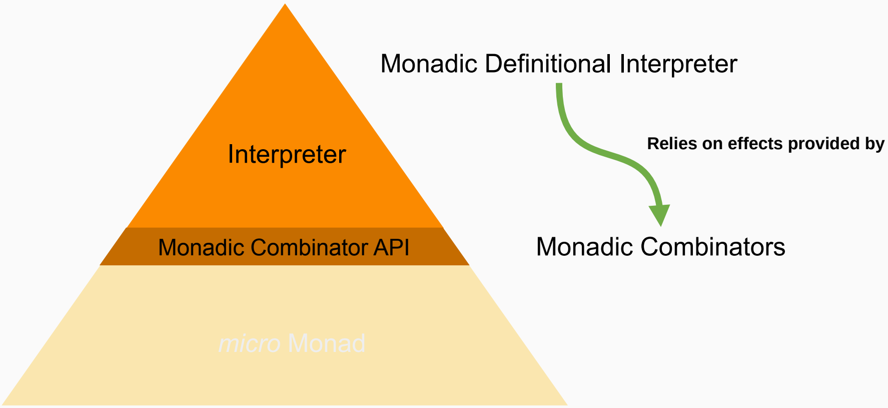
```
perform e

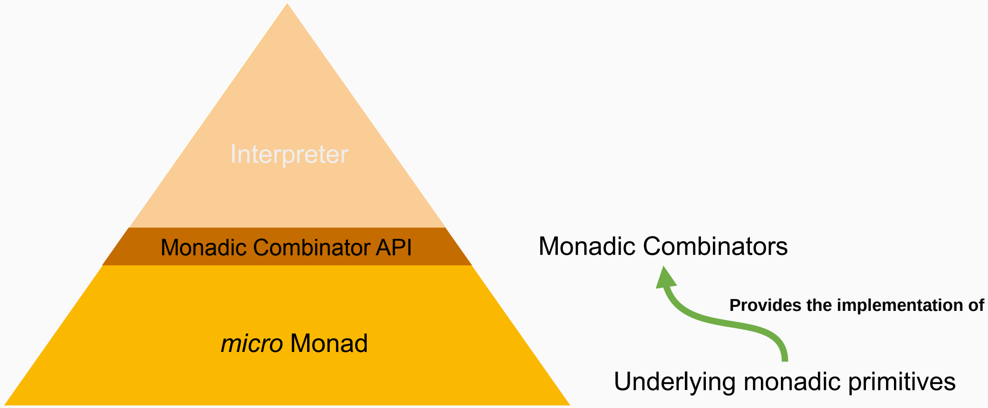
match e with
| p₁ -> e₁
| exception p₂ -> e₂
| effect p₃, k -> e₃

continue k e

discontinue k e
```

```
Fixpoint eval η e :=
  | ...
  | EPerform e =>
    eff <- eval η e ;
    perform eff
  | EMatch e bs =>
    handle (eval η e) (fun o => eval_branches η o bs)
  | EContinue e1 e2 =>
    k <- as_cont (eval η e1) ;
    v <- eval η e2 ;
    resume k (O2Ret v)
  | EDiscontinue e1 e2 =>
    k <- as_cont (eval η e1) ;
    v <- eval η e2 ;
    resume k (O2Throw v)
```


Interpreter
Monadic Combinator API
micro Monad
Monadic Definitional Interpreter
Relies on effects provided by
Monadic Combinators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Monadic Combinator API
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Monadic Combinators
Provides the implementation of
Underlying monadic primitives

A Syntax for Computations

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$$\begin{aligned} please\_eval\ \eta\ e &:= \textcolor{orange}{Stop}\ CEval\,(\eta, e)\ id_2 \\ perform\ v &:= \textcolor{orange}{Stop}\ CPerf\ v\ id_2 \\ resume\ k\ v &:= \textcolor{orange}{Stop}\ CResume\,(k, v)\ id_2 \end{aligned}$$

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\end{aligned}
$$

$$
\boxed{
\begin{aligned}
&\mathit{id}_2 : \mathit{outcome}_2 \to \mathit{micro}\ \mathit{val}\ \mathit{exn} \\
&\mathit{id}_2\ (\mathit{O2Ret}\ v) = \mathit{Ret}\ v \\
&\mathit{id}_2\ (\mathit{O2Throw}\ \mathit{ex}) = \mathit{Throw}\ \mathit{ex}
\end{aligned}
}
$$

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\end{aligned}
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- *Ret* is a leaf;
- A *Stop* node represents a "system call" and carries one child for each possible result;

Ret and *Stop* alone form the freer monad (Kiselyov & Ishii, 2015).

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$$\begin{aligned} par\ m_1\ m_2 &:= Par\ m_1\ m_2\ id_2 \\ handle\ m\ h &:= Handle\ m\ h \end{aligned}$$

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- *Ret*, *Throw*, *Crash* are leaves;
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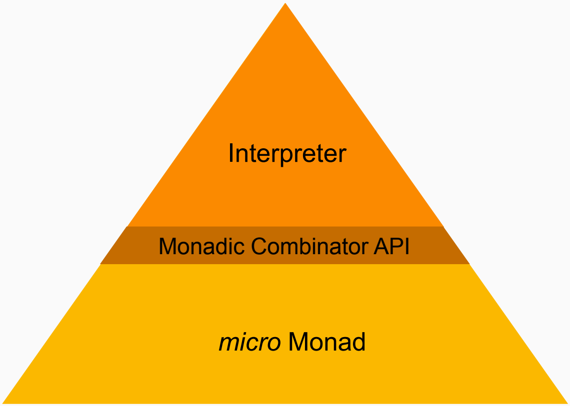
$$
\begin{aligned}
&\textsf{Inductive}\ \mathit{micro}\ A\ E :=\\
&\quad \mid \textbf{Ret} &&: A \to \mathit{micro}\ A\ E\\
&\quad \mid \textbf{Throw} &&: E \to \mathit{micro}\ A\ E\\
&\quad \mid \textbf{Crash} &&: \mathit{micro}\ A\ E\\
&\quad \mid \textbf{Stop} &&: \mathit{code}\ X\ Y\ E' \to X \to (\mathit{outcome}_2\ Y\ E' \to \mathit{micro}\ A\ E) \to \mathit{micro}\ A\ E\\
&\quad \mid \textbf{Par} &&: \mathit{micro}\ A_1\ E' \to \mathit{micro}\ A_2\ E' \to\\
& && \qquad (\mathit{outcome}_2\ (A_1 \times A_2)\ E' \to \mathit{micro}\ A\ E) \to \mathit{micro}\ A\ E\\
&\quad \mid \textbf{Handle} &&: \mathit{micro}\ \mathit{val}\ \mathit{exn} \to (\mathit{outcome}_3\ \mathit{val}\ \mathit{exn} \to \mathit{micro}\ A\ E) \to \mathit{micro}\ A\ E
\end{aligned}
$$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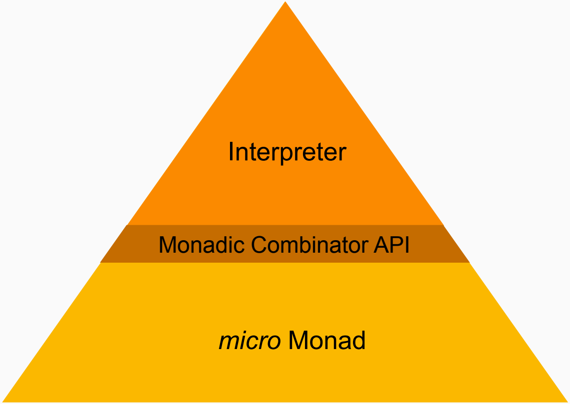

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$\longrightarrow$

A Small-Step Reduction Semantics

The meaning, or *behaviour*, of an effect is given by a *small-step semantics*.

$$m \;/\; \sigma \longrightarrow m' \;/\; \sigma'$$

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$$m \;/\; \sigma \longrightarrow m' \;/\; \sigma'$$

The system call *CEval* reduces to a recursive call to *eval*.

$$\mathit{Stop}\;\mathit{CEval}\;(\eta, e)\;k \;/\; \sigma \longrightarrow \mathit{try}_2\;(\mathit{eval}\;\eta\;e)\;k \;/\; \sigma$$

This technique is inspired by *Turing Completeness Completely Free*, McBride (2015).

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$$please\_eval \ \eta \ e \ / \ \sigma \longrightarrow eval \ \eta \ e \ / \ \sigma$$

This technique is inspired by *Turing Completeness Completely Free*, McBride (2015).

Delimited Control

Handle observes a computation's $outcome_3$ and invokes a handler.

$$\frac{m \;/\; \sigma \longrightarrow m' \;/\; \sigma'}{handle\; m\; h \;/\; \sigma \longrightarrow handle\; m'\; h \;/\; \sigma'}$$

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$$\begin{array}{rcl}
handle\,(Ret\ \ v)\ h\,/\,\sigma & \longrightarrow & h\ (O3Ret\ \ v)\,/\,\sigma \\
handle\,(Throw\ \ ex)\ h\,/\,\sigma & \longrightarrow & h\ (O3Throw\ \ ex)\,/\,\sigma \\
handle\,(Stop\ CPerf\ \ v\ k)\ h\,/\,\sigma & \longrightarrow & h\ (Perform_3\ \ v\ \ell)\,/\,\sigma[\ell := k] \text{ with } \ell \text{ fresh}
\end{array}$$

Delimited Control

Handle observes a computation's *outcome*$_3$ and invokes a handler.

$$\frac{m \;/\; \sigma \longrightarrow m' \;/\; \sigma'}{\mathit{handle}\ m\ h \;/\; \sigma \longrightarrow \mathit{handle}\ m'\ h \;/\; \sigma'}$$

$$\begin{array}{rcl}
\mathit{handle}\,(\mathit{Ret}\ v)\ h \,/\, \sigma & \longrightarrow & h\,(\mathit{O3Ret}\ v) \,/\, \sigma \\
\mathit{handle}\,(\mathit{Throw}\ \mathit{ex})\ h \,/\, \sigma & \longrightarrow & h\,(\mathit{O3Throw}\ \mathit{ex}) \,/\, \sigma \\
\mathit{handle}\,(\mathit{Stop}\ \mathit{CPerf}\ v\ k)\ h \,/\, \sigma & \longrightarrow & h\,(\mathit{Perform}_3\ v\ \ell) \,/\, \sigma[\ell := k] \text{ with } \ell \text{ fresh}
\end{array}$$

A *heap* σ maps memory locations to values (v) or continuations (k or $\lightning$).

Delimited Control

The system call *CResume* fetches and resumes a stored continuation.

$$\frac{\sigma(\ell) = k}{\mathit{resume}\ \ell\ o\ /\ \sigma \longrightarrow k\ o\ /\ \sigma[\ell := \lightning]}$$

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$$\frac{\sigma(\ell) = k}{\mathit{resume}\ \ell\ o \ /\ \sigma \longrightarrow k\ o \ /\ \sigma[\ell := \lightning]}$$

$$\frac{\sigma(\ell) = \lightning}{\mathit{resume}\ \ell\ o \ /\ \sigma \longrightarrow \mathit{Crash} \ /\ \sigma}$$

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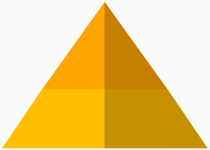
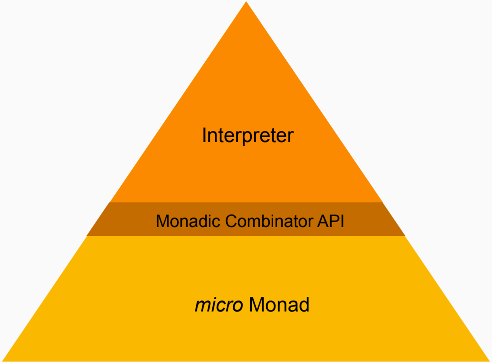


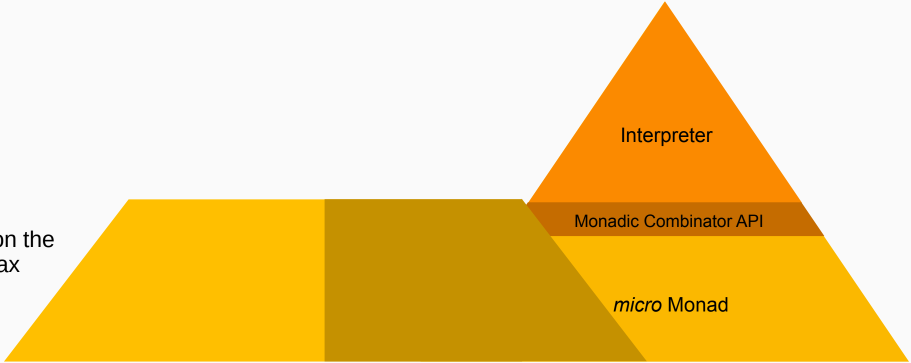
Building our Program Logics

$\longrightarrow$ **+**



Building our Program Logics

Reasoning rules on the
monadic syntax



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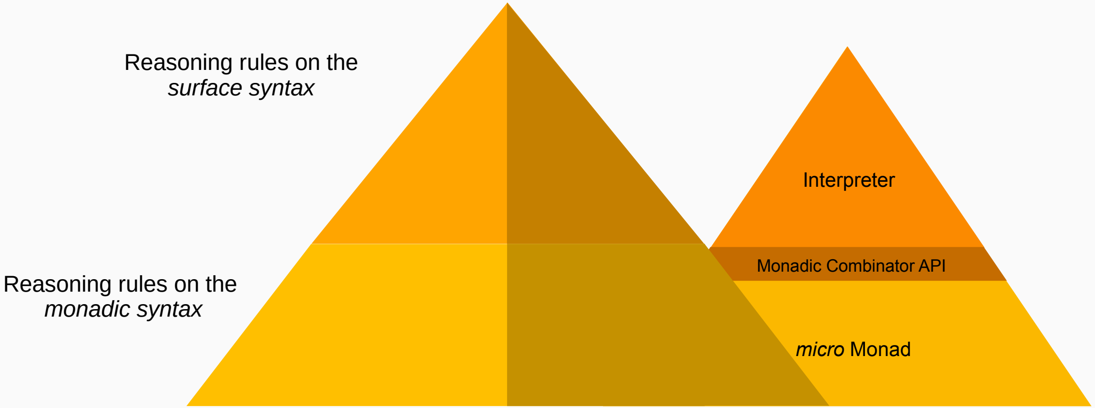

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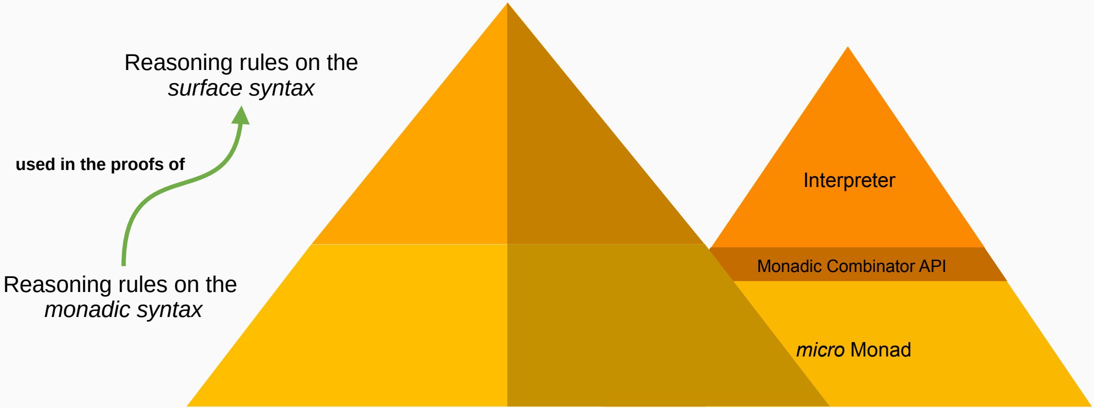
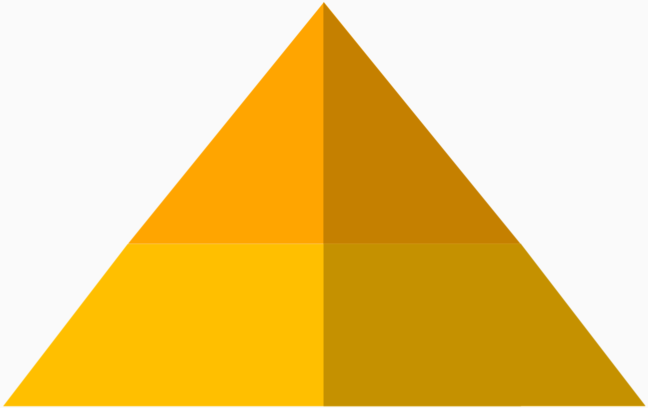


Horus: Hoare-Style Reasoning About Pure Programs



Horus: Hoare-Style Reasoning About Pure Programs

pure m φ ζ

Horus: Hoare-Style Reasoning About Pure Programs

$$\boxed{\begin{array}{l} \varphi : \mathsf{val} \to \mathsf{Prop} \\ \zeta : \mathsf{exn} \to \mathsf{Prop} \end{array}}$$

$$\mathit{pure}\ m\ \varphi\ \zeta$$

Horus: Hoare-Style Reasoning About Pure Programs

$$\boxed{\begin{aligned}\varphi &: \mathsf{val} \to \mathsf{Prop}\\ \zeta &: \mathsf{exn} \to \mathsf{Prop}\end{aligned}}$$

$$\mathit{pure}\ m\ \varphi\ \zeta$$

$$\mathit{pure}\ (\mathit{please\_eval}\ \eta\ e)\ \varphi\ \zeta$$

Horus: Hoare-Style Reasoning About Pure Programs

$$\boxed{\begin{array}{l}\varphi \ : \ \mathsf{val} \to \mathsf{Prop} \\ \zeta \ : \ \mathsf{exn} \to \mathsf{Prop}\end{array}}$$

$$\mathit{pure}\ m\ \varphi\ \zeta$$

$$\mathit{pure}\ (\mathit{eval}\ \eta\ e)\ \varphi\ \zeta$$

$$\mathit{pure}\ (\mathit{eval\_branches}\ \eta\ o\ \mathit{bs})\ \varphi\ \zeta$$

$$\mathit{pure}\ (\mathit{please\_eval}\ \eta\ e)\ \varphi\ \zeta$$

Horus: Hoare-Style Reasoning About Pure Programs

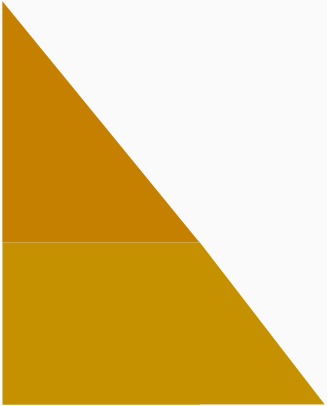
$$\mathit{expr}\ \eta\ e\ \varphi\ \zeta$$

$$\mathit{branches}\ \eta\ o\ \mathit{bs}\ \varphi\ \zeta$$

$$\mathit{pure}\ m\ \varphi\ \zeta$$

$$\mathit{pure}\ (\mathit{please\_eval}\ \eta\ e)\ \varphi\ \zeta$$

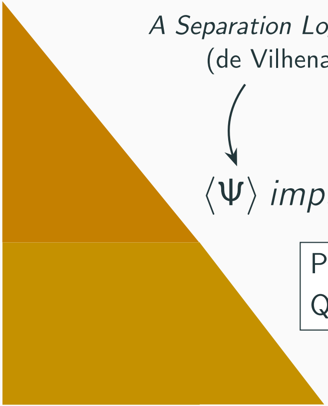
Iris-Style Reasoning About Arbitrary Programs



Iris-Style Reasoning About Arbitrary Programs

$\langle \Psi \rangle$ *impure m P Q*

Iris-Style Reasoning About Arbitrary Programs



A Separation Logic for Effect Handlers,
(de Vilhena & Pottier, 2021)

$$\langle \Psi \rangle \; \mathit{impure} \; m \; P \; Q$$

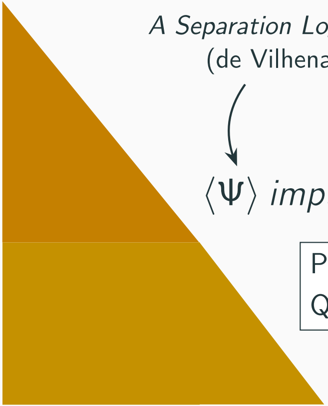
$$\boxed{\begin{array}{l} \mathsf{P} : \mathsf{val} \to \mathsf{iProp} \\ \mathsf{Q} : \mathsf{exn} \to \mathsf{iProp} \end{array}}$$

Iris-Style Reasoning About Arbitrary Programs

A Separation Logic for Effect Handlers, (de Vilhena & Pottier, 2021)

$\langle\Psi\rangle$ *impure* m P Q

$$\boxed{\begin{array}{l} \mathsf{P} : \mathsf{val} \to \mathsf{iProp} \\ \mathsf{Q} : \mathsf{exn} \to \mathsf{iProp} \end{array}}$$



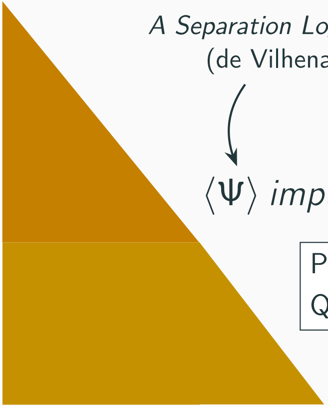
$\langle\Psi\rangle$ *impure* $(handle\ m\ h)$ P Q

Iris-Style Reasoning About Arbitrary Programs

$\langle \Psi \rangle$ *impure* $(\mathit{eval}\ \eta\ e)\ P\ Q$

deep-handler $\Psi\ P\ Q\ \eta\ o\ \mathit{bs}\ P'\ Q'$

$\langle \Psi \rangle$ *impure* $(\mathit{handle}\ m\ h)\ P\ Q$



A Separation Logic for Effect Handlers, (de Vilhena & Pottier, 2021)

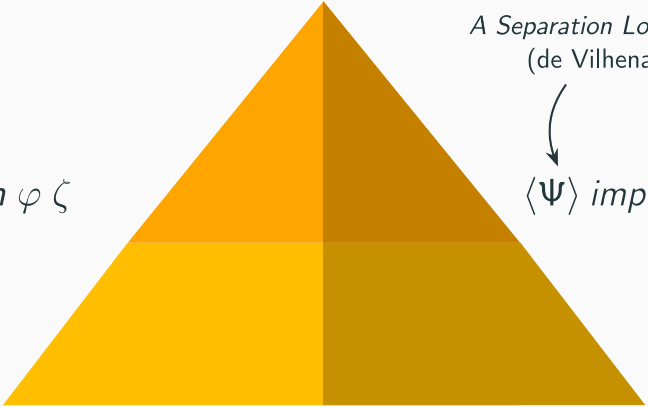
$\langle \Psi \rangle$ *impure* $m\ P\ Q$

$\boxed{\begin{array}{l} \mathsf{P} : \mathsf{val} \rightarrow \mathsf{iProp} \\ \mathsf{Q} : \mathsf{exn} \rightarrow \mathsf{iProp} \end{array}}$

A Separation Logic for Effect Handlers, (de Vilhena & Pottier, 2021)

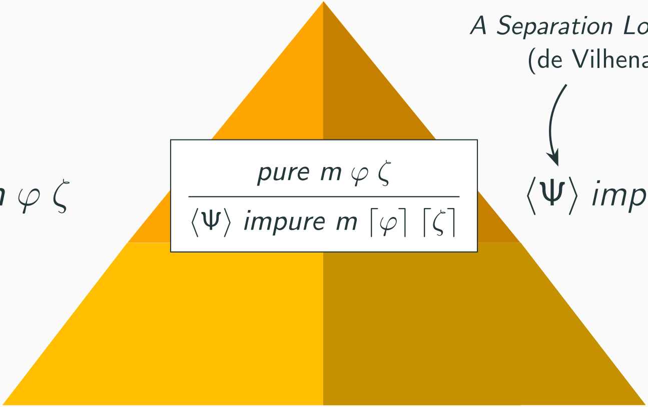
$\langle \Psi \rangle$ *impure* $m\ P\ Q$

pure $m\ \varphi\ \zeta$



Iris-Style Reasoning About Arbitrary Programs

$\mathit{pure}\ m\ \varphi\ \zeta$

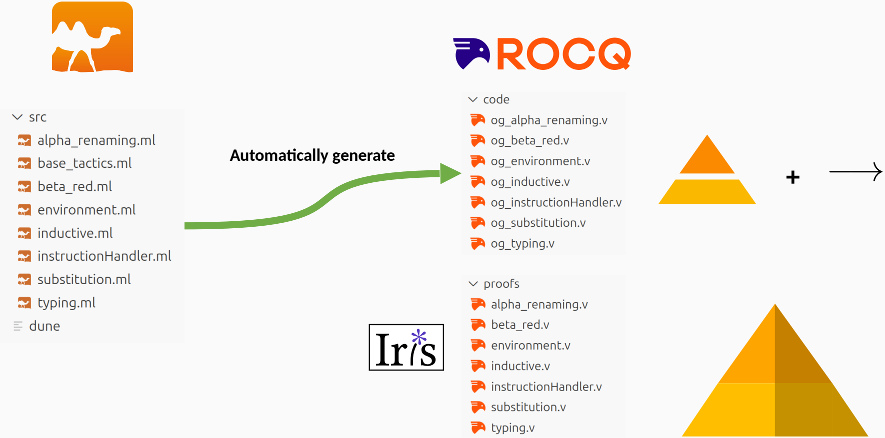


$$\frac{\mathit{pure}\ m\ \varphi\ \zeta}{\langle\Psi\rangle\ \mathit{impure}\ m\ \lceil\varphi\rceil\ \lceil\zeta\rceil}$$

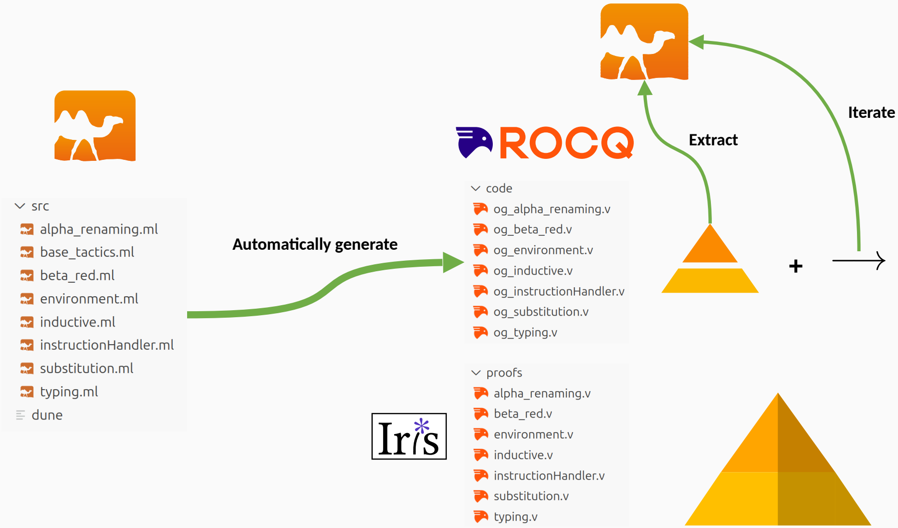
A Separation Logic for Effect Handlers,
(de Vilhena & Pottier, 2021)

$\langle\Psi\rangle\ \mathit{impure}\ m\ P\ Q$

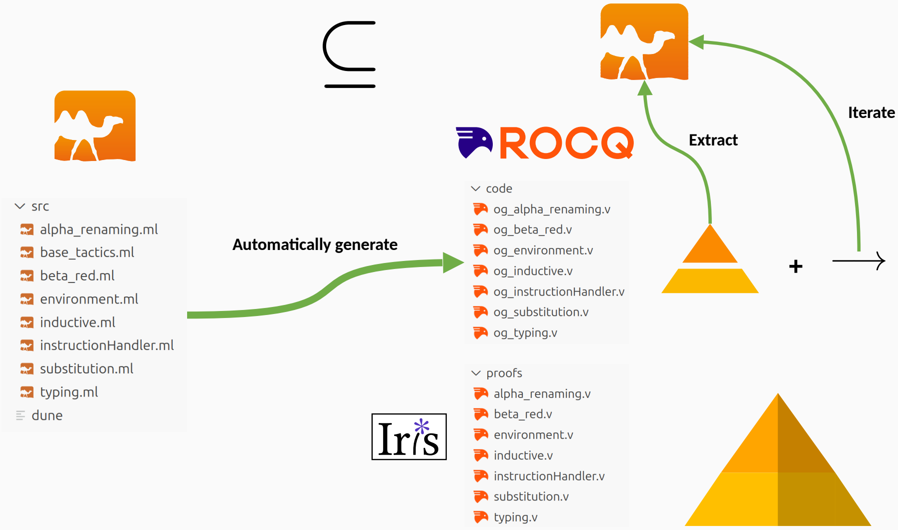
Validating the Semantics

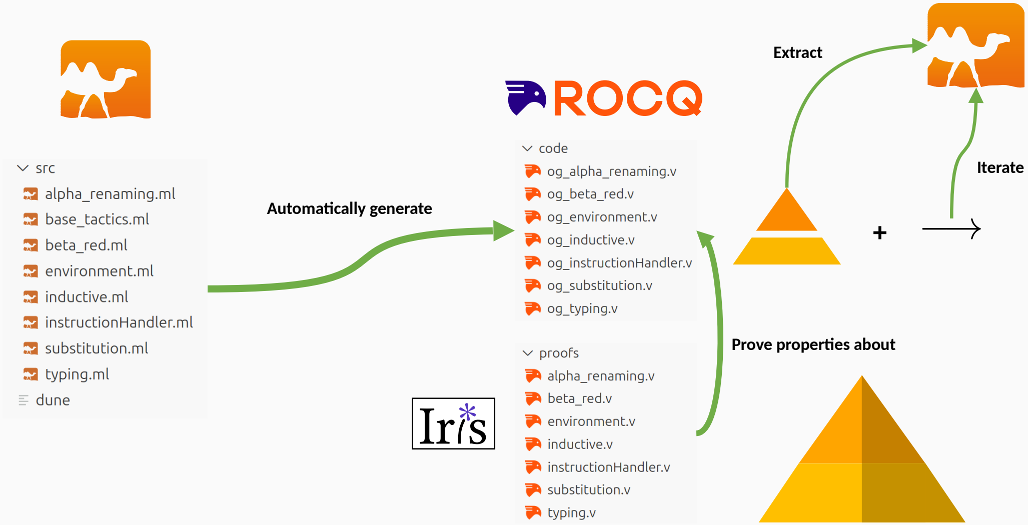


Validating the Semantics

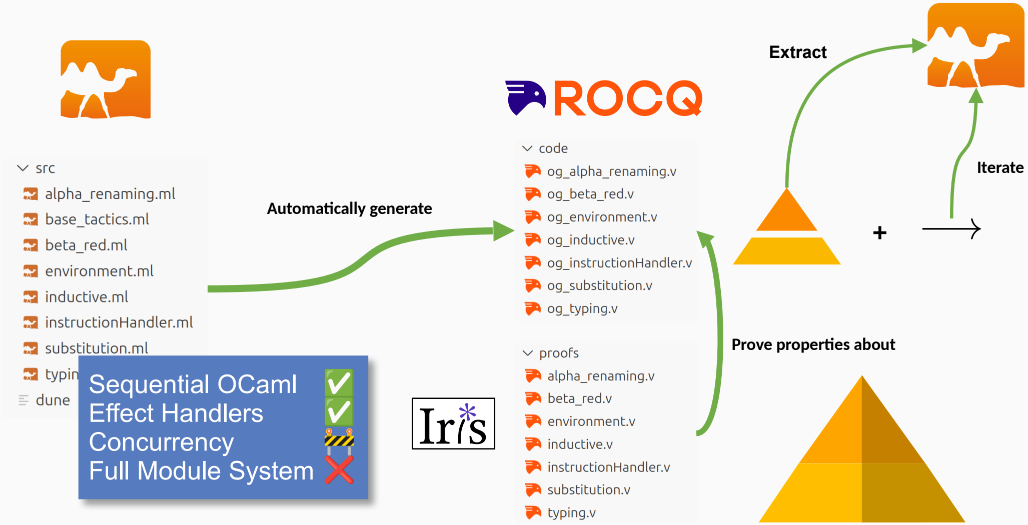


Validating the Semantics





src
alpha_renaming.ml
base_tactics.ml
beta_red.ml
environment.ml
inductive.ml
instructionHandler.ml
substitution.ml
typing.ml
dune
Automatically generate
ROCQ
code
og_alpha_renaming.v
og_beta_red.v
og_environment.v
og_inductive.v
og_instructionHandler.v
og_substitution.v
og_typing.v
proofs
alpha_renaming.v
beta_red.v
environment.v
inductive.v
instructionHandler.v
substitution.v
typing.v
Iris
Extract
Iterate
+
Prove properties about


src
alpha_renaming.ml
base_tactics.ml
beta_red.ml
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substitution.ml
typin
dune
Sequential OCaml ✅
Effect Handlers ✅
Concurrency 🚧
Full Module System ❌
Automatically generate
Iris
ROCQ
code
og_alpha_renaming.v
og_beta_red.v
og_environment.v
og_inductive.v
og_instructionHandler.v
og_substitution.v
og_typing.v
proofs
alpha_renaming.v
beta_red.v
environment.v
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Extract
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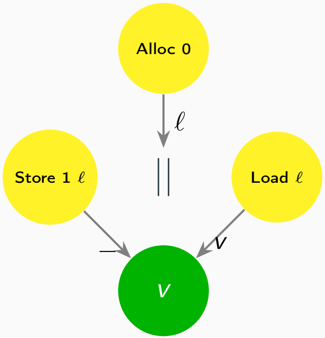
Prove properties about

Backup Slides

Nondeterminism / Parallelism

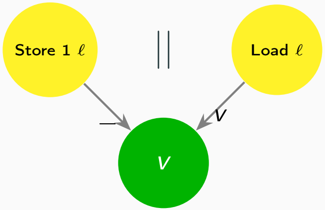
```
let x = ref 0 in
let ((), v) = ((x := 1), !x) in
v
```



Nondeterminism / Parallelism

```
let x = ref 0 in
let ((), v) = ((x := 1), !x) in
v
```

$\boxed{\ell \mapsto 0}$



Nondeterminism / Parallelism

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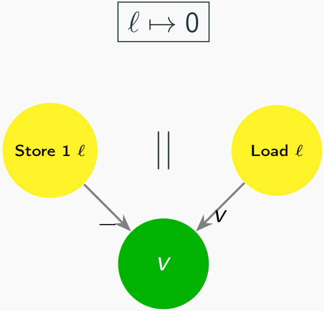

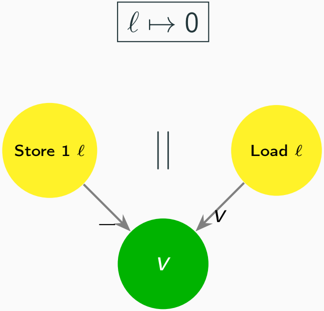

Nondeterminism / Parallelism

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Nondeterminism / Parallelism

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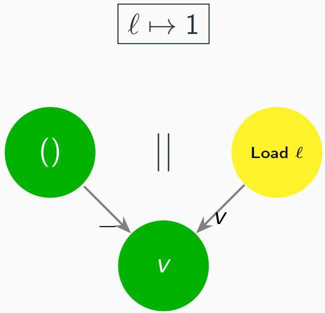

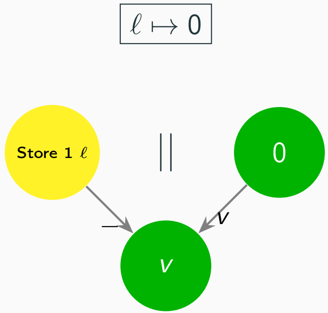

Nondeterminism / Parallelism

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let x = ref 0 in
let ((), v) = ((x := 1), !x) in
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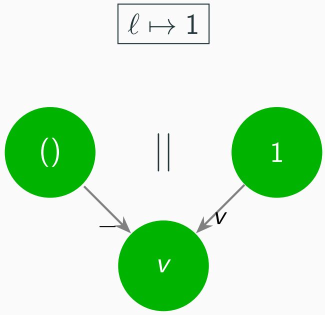

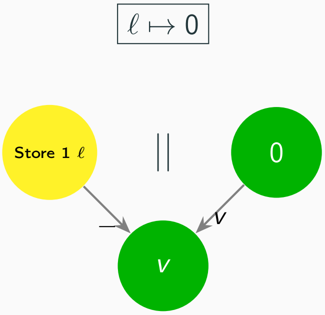

Nondeterminism / Parallelism

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let x = ref 0 in
let ((), v) = ((x := 1), !x) in
v
```

Nondeterminism / Parallelism

```
let x = ref 0 in
let ((), v) = ((x := 1), !x) in
v
```

$\boxed{\ell \mapsto 1}$ $\boxed{\ell \mapsto 1}$

1 0

```
let x = ref 0 in
let ((), v) = ((x := 1), !x) in
v
```

$\boxed{\ell \mapsto 1}$

1

```
let x = ref 0 in
let (_, v) = ((x := 1), !x) in
v
```

$\boxed{\ell \mapsto 1}$

0

Shallow Effect Handlers

Supporting shallow effect handlers only requires removing the instance of *wrap* in *eval_branches*.

```
Fixpoint eval η e :=
  ...
  | EShallowMatch e bs ⇒
      handle (eval η e) (λ o ⇒ shallow_eval_branches η o bs)
```

Shallow matches have a different set of proof rules, but they are built atop of the same *EWP*.

The system call *CWrap* wraps a stored continuation in an effect handler, yielding a new stored continuation.

$$\begin{aligned} \mathit{Stop}\ \mathit{CWrap}\,(\eta, \ell, \mathit{bs})\ /\ \sigma \quad \longrightarrow \quad & \mathit{O2Ret}\ \ell'\ /\ \sigma[\ell' := k'] \\ & \text{if } \ell' \notin \mathit{dom}(\sigma) \end{aligned}$$

Delimited Control

The system call *CWrap* wraps a stored continuation in an effect handler, yielding a new stored continuation.

$$\begin{array}{lll} \mathit{Stop}\ \mathit{CWrap}\,(\eta, \ell, \mathit{bs})\ /\ \sigma & \longrightarrow & \mathit{O2Ret}\ \ell'\ /\ \sigma[\ell' := k'] \\ & & \quad \text{if } \ell' \notin \mathit{dom}(\sigma) \end{array}$$

$$\text{where } k' = \lambda o.\, \mathit{Handle}\ (\mathit{resume}\ \ell\ o)\ (\lambda o.\, \mathit{eval\_branches}\ \eta\ o\ \mathit{bs})$$